Effect of Concentration on Particle Drag

Section (Neb) reviewed the dependence of the drag coefficient on the Reynolds number for a single particle in a fluid and the effect on the sedimentation of that single particle in an otherwise quiescent fluid was examined as a particular example. Such results would be directly applicable to the evaluation of the relative velocity between the disperse phase (the particles) and the continuous phase in a very dilute multiphase flow. However, at higher concentrations, the interactions between the flow fields around individual particles alter the force experienced by those particles and therefore change the velocity of sedimentation. Furthermore, the volumetric flux of the disperse phase is no longer negligible because of the finite concentration and, depending on the boundary conditions in the particular problem, this may cause a non-negligible volumetric flux of the continuous phase. For example, particles sedimenting in a containing vessel with a downward particle volume flux, $-j_S$ (upward is deemed the positive direction), at a concentration, α, will have a mean velocity,

$$-u_S = -j_S/\alpha \qquad \text{(Nel1)}$$

and will cause an equal and opposite upward flux of the suspending liquid, $j_L = -j_S$, so that the mean velocity of the liquid,

$$u_L = j_L/(1-\alpha) = -j_S/(1-\alpha) \qquad \text{(Nel2)}$$

Hence the relative velocity is

$$u_{SL} = u_S - u_L = j_S/\alpha(1-\alpha) = u_S/(1-\alpha) \qquad \text{(Nel3)}$$

Thus care must be taken to define the *terminal velocity* and here we shall focus on the more fundamental quantity, namely the relative velocity, u_{SL}, rather than quantities such as the sedimentation velocity, u_S, that are dependent on the boundary conditions.

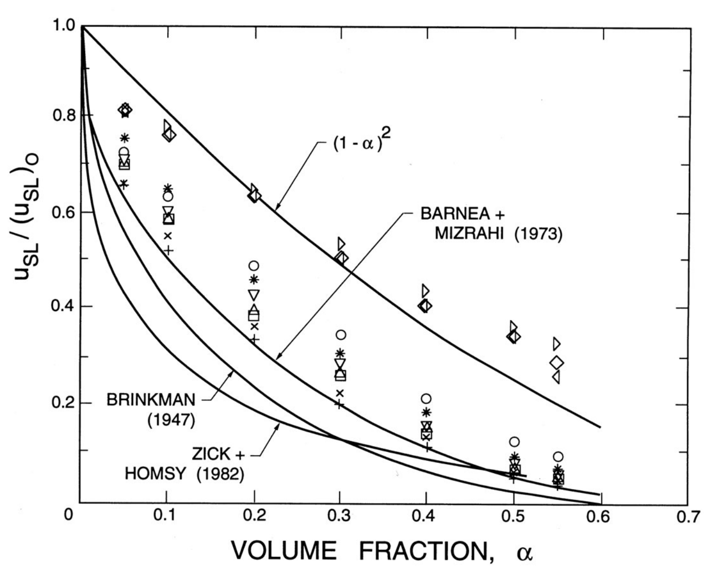

Figure 1: Relative velocity of sedimenting particles, u_{SL} (normalized by the velocity as $\alpha \to 0$, $(u_{SL})_0$) as a function of the volume fraction, α. Experimental data from Mertes and Rhodes (1955) are shown for various Reynolds numbers, Re, as follows: $Re = 0.003$ (+), 0.019 (×), 0.155 (⊡), 0.98 (△), 1.45 (▽), 4.8 (∗), 16 (⊙), 641 (⋄), 1020 (▷) and 2180 (◁). Also shown are the analytical results of Brinkman (equation (Nel5)) and Zick and Homsy and the empirical results of Wallis (equation (Nel8)) and Barnea and Mizrahi (equation (Nel6)).

Barnea and Mizrahi (1973) have reviewed the experimental, theoretical and empirical data on the sedimentation of particles in otherwise quiescent fluids at various concentrations, α. The experimental data of Mertes and Rhodes (1955) on the ratio of the relative velocity, u_{SL}, to the sedimentation velocity for a single particle, $(u_{SL})_0$ (equal to the value of u_{SL} as $\alpha \rightarrow 0$), are presented in figure 1. As one might anticipate, the relative motion is hindered by the increasing concentration. It can also be seen that $u_{SL}/(u_{SL})_0$ is not only a function of α but varies systematically with the Reynolds number, $2R(u_{SL})_0/\nu_L$, where ν_L is the kinematic viscosity of the suspending medium. Specifically, $u_{SL}/(u_{SL})_0$ increases significantly with Re so that the rate of decrease of $u_{SL}/(u_{SL})_0$ with increasing α is lessened as the Reynolds number increases. One might intuitively expect this decrease in the interactions between the particles since the far field effects of the flow around a single particle decline as the Reynolds number increases.

We also note that complementary to the data of figure 1 is extensive data on the flow through packed beds of particles. The classical analyses of that data by Kozeny (1927) and, independently, by Carman (1937) led to the widely used expression for the pressure drop in the low Reynolds number flow of a fluid of viscosity, μ_C, and superficial velocity, j_{CD}, through a packed bed of spheres of diameter, D, and solids volume fraction, α, namely:

$$\frac{dp}{ds} = \frac{180\alpha^3 \mu_C j_{CD}}{(1-\alpha)^3 D^2} \qquad \text{(Nel4)}$$

where the 180 and the powers on the functions of α were empirically determined. This expression, known as the Carman-Kozeny equation, will be used shortly.

Several curves that are representative of the analytical and empirical results are also shown in figure 1 (and in figure 2). One of the first approximate, analytical models to include the interactions between particles was that of Brinkman (1947) for spherical particles at asymptotically small Reynolds numbers who obtained

$$\frac{u_{SL}}{(u_{SL})_0} = \frac{(2-3\alpha)^2}{4+3\alpha+3(8\alpha-3\alpha^2)^{\frac{1}{2}}} \qquad \text{(Nel5)}$$

and this result is included in figures 1 and 2. Other researchers (see, for example, Tam 1969 and Brady and Bossis 1988) have studied this low Reynolds number limit quite closely. Exact solutions for the sedimentation velocity of a various regular arrays of spheres at asymptotically low Reynolds number were obtained by Zick and Homsy (1982) and the particular result for a simple cubic array is included in figure 1. Clearly, these results deviate significantly from the experimental data and it is currently thought that the sedimentation process cannot be modeled by a regular array because the fluid mechanical effects are dominated by the events that occur when particles happen to come close to one another.

Switching attention to particle Reynolds numbers greater than unity, it was mentioned earlier that the work of Fortes *et al.* (1987) and others has illustrated that the interactions between particles become very complex since they result, primarily, from the interactions of particles with the wakes of the particles ahead of them. Fortes *et al.* (1987) have shown this results in a variety of behaviors they term *drafting*, *kissing* and *tumbling* that can be recognized in fluidized beds. As yet, these behaviors have not been amenable to theoretical analyses.

The literature contains numerous empirical correlations but three will suffice for present purposes. At small Reynolds numbers, Barnea and Mizrahi (1973) show that the experimental data closely follow an expression of the form

$$\frac{u_{SL}}{(u_{SL})_0} \approx \frac{(1-\alpha)}{(1+\alpha^{\frac{1}{3}})e^{5\alpha/3(1-\alpha)}} \qquad \text{(Nel6)}$$

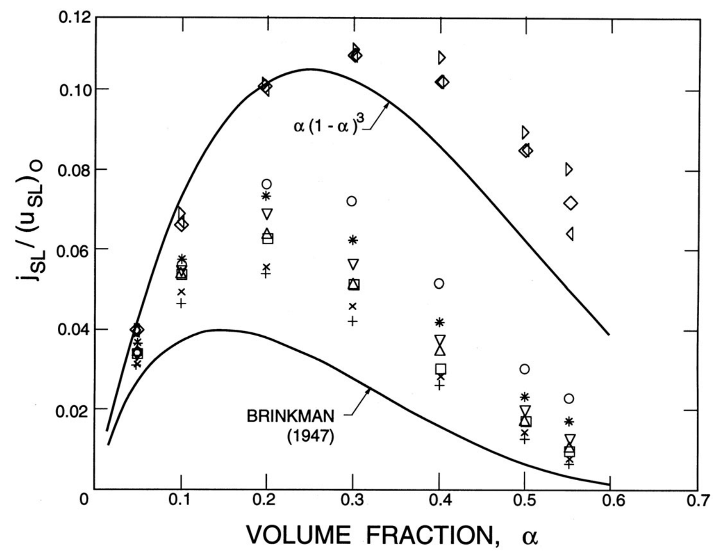

Figure 2: The drift flux, j_{SL} (normalized by the velocity $(u_{SL})_0$) corresponding to the relative velocities of figure 1 (see that caption for codes).

By way of comparison the Carman-Kozeny equation (Nel4) implies that a sedimenting packed bed would have a terminal velocity given by

$$\frac{u_{SL}}{(u_{SL})_0} = \frac{1}{80}\frac{(1-\alpha)^2}{\alpha^2} \tag{Nel7}$$

which has magnitudes comparable to the expression (Nel6) at the volume fractions of packed beds.

At large rather than small Reynolds numbers, the ratio $u_{SL}/(u_{SL})_0$ seems to be better approximated by the empirical relation

$$\frac{u_{SL}}{(u_{SL})_0} \approx (1-\alpha)^{b-1} \tag{Nel8}$$

where Wallis (1969) suggests a value of $b = 3$. Both of these empirical formulae are included in figure 1.

In later sections discussing sedimentation phenomena, we shall use the drift flux, j_{SL}, more frequently than the relative velocity, u_{SL}. Recalling that, $j_{SL} = \alpha(1-\alpha)u_{SL}$, the data from figure 1 are replotted in figure 2 to display $j_{SL}/(u_{SL})_0$.

It is appropriate to end by expressing some reservations regarding the generality of the experimental data presented in figures 1 and 2. At the higher concentrations, vertical flows of this type often develop instabilities that produce large scale mixing motions whose scale is of the same order as the horizontal extent of the flow, usually the pipe or container diameter. In turn, these motions can have a substantial effect on the mean sedimentation velocity. Consequently, one might expect a pipe size effect that would manifest itself non-dimensionally as a dependence on a parameter such as the ratio of the particle to pipe diameter, $2R/d$, or, perhaps, in a Froude number such as $(u_{SL})_0/(gd)^{\frac{1}{2}}$. Another source of discrepancy could be a dependence on the overall flow rate. Almost all of the data, including that of Mertes and Rhodes (1955), has been obtained from relatively quiescent sedimentation or fluidized bed experiments in which the overall flow rate is small and, therefore, the level of turbulence is limited to that produced by the relative motion between the particles and the suspending fluid. However, when the overall flow rate is increased so that even a single phase flow of the suspending fluid would be turbulent, the mean sedimentation velocities may be significantly altered by the enhancement of the mixing and turbulent motions. Figure 3 presents data from some experiments by Bernier (1982) in which the relative velocity of bubbles of air in a vertical

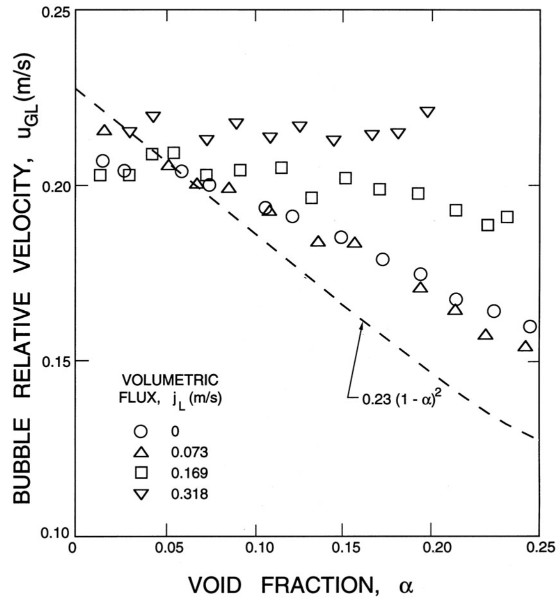

Figure 3: Data indicating the variation in the bubble relative velocity, u_{GL}, with the void fraction, α, and the overall flow rate (as represented by j_L) in a vertical, $10.2cm$ diameter tube. The dashed line is the correlation of Wallis, equation (Nel8). Adapted from Bernier (1982).

water flow were measured for various total volumetric fluxes, j. Small j values cause little deviation from the behavior at $j = 0$ and are consistent with the results of figure 1. However, at larger j values for which a single phase flow would be turbulent, the decrease in u_{GL} with increasing α almost completely disappears. Bernier surmised that this disappearance of the interaction effect is due to the increase in the turbulence level in the flow that essentially overwhelms any particle/particle or bubble/bubble interaction.