3.6.1 Diffusion theory

It is appropriate to recall at this point that diffusion theory for the neutronics of a reactor core avoids much complexity posed by the interior structure of the reactor core by assuming:

1. that the reactor core can be considered to be homogeneous. As previously described this requires the assumption that the neutron mean free paths are long compared with the typical small-scale interior dimensions of the reactor core (such as the fuel rod dimensions). This then allows characterization of the dynamics by a single neutron flux, ϕ, though one that varies with time and from place to place. Fuel rods are typically only a few *cm* in diameter and with neutron diffusion lengths, L (see equation 3.19), of about 60 *cm* this criterion is crudely satisfied in most thermal reactors.

2. that the characteristic neutron flux does not vary substantially over one mean free path. This is known as a *weakly absorbing* medium.

3. that the reactor core is large compared with the neutron mean free paths so that a neutron will generally experience many interactions within the core before encountering one of the core boundaries. Most thermal reactor cores are only a few neutron diffusion lengths, L, in typical dimension so this criterion is only very crudely satisfied.

These last two assumptions effectively mean that neutrons diffuse within the core and the overall population variations can be characterized by a diffusion equation.

In addition to the governing equation, it is necessary to establish both initial conditions and boundary conditions on the neutron flux, ϕ. Initial conditions will be simply given by some known neutron flux, $\phi(x_i, 0)$, at the initial time, $t = 0$. The evaluation of boundary conditions requires the development of relations for the one-way flux of neutrons through a surface or discontinuity. To establish such relations the one-way flux of neutrons through any surface or boundary (the coordinate x_n is defined as normal to this boundary in the positive direction) will be denoted by J_n^+ in the positive direction and by J_n^- in the negative direction. Clearly the net flux of neutrons will be equal to J_n so that

$$J_n^+ - J_n^- = J_n = -D\frac{\partial \phi}{\partial x_n} \tag{1}$$

using equations 3.11 and 3.12. On the other hand the sum of these same two fluxes must be related to the neutron flux; specifically

$$J_n^+ + J_n^- = \frac{1}{2}\phi \tag{2}$$

(see, for example, Glasstone and Sesonske 1981). The factor of one half is geometric: since the flux ϕ is in all directions, the resultant in the direction x_n requires the average value of the cosine of the angle relative to x_n.

It follows from equations 1 and 2 that

$$J_n^+ = \frac{1}{4}\phi - \frac{D}{2}\frac{\partial \phi}{\partial x_n} \qquad ; \qquad J_n^- = \frac{1}{4}\phi + \frac{D}{2}\frac{\partial \phi}{\partial x_n} \tag{3}$$

These relations allow the establishment of boundary conditions when the condition involves some constraint on the neutron flux. Two examples will suffice.

At an interface between two different media denoted by subscripts 1 and 2 (and with diffusion coefficients D_1 and D_2), the neutron flux into medium 1 must be equal to the neutron flux out of medium 2 and, conversely, the neutron flux out of medium 1 must be equal to the neutron flux into medium 2. Therefore from equations 3 it follows that, at the interface:

$$\phi_1 = \phi_2 \qquad \text{and} \qquad D_1\frac{\partial \phi_1}{\partial x_n} = D_2\frac{\partial \phi_2}{\partial x_n} \tag{4}$$

A second, practical example is the boundary between one medium (subscript 1) and a vacuum from which there will be no neutron flux back into the first medium. This is an approximation to the condition at the surface boundary of a reactor. Then on that boundary it is clear that $J_n^- = 0$ where x_n is in the direction of the vacuum. Then it follows that at the boundary

$$\phi_1 = -2D\frac{\partial \phi_1}{\partial x_n} \tag{5}$$

One way to implement this numerically is to use a linear extrapolation and set ϕ_1 to be zero at a displaced, virtual boundary that is a distance $1/2D$ into the vacuum from the actual boundary. This displacement, $1/2D$, is known as the *linear extrapolation length*.