6.2.6 Limits on separated flow

Attention will now be turned to the limits on separated flow regimes and the primary mechanism that determines that limit is the Kelvin-Helmholtz instability. Separated flow regimes such as stratified horizontal flow or vertical annular flow can become unstable when waves form on the interface between the two fluid streams (subscripts 1 and 2). As indicated in figure 1, the densities of the fluids will be denoted by ρ_1 and ρ_2 and the velocities by u_1 and u_2. If these waves continue to grow in amplitude they cause a transition to another flow regime, typically one with greater intermittency and involving plugs or slugs. Therefore, in order to determine this particular boundary of the separated flow regime, it is necessary to investigate the potential growth of the interfacial waves, whose wavelength will be denoted by λ (wavenumber, $\kappa = 2\pi/\lambda$). Studies of such waves have a long history originating with the work of Kelvin and Helmholtz and the phenomena they revealed have come to be called Kelvin-Helmholtz instabilities (see, for example, Yih 1965). In general this class of instabilities involves the interplay between at least two of the following three types of forces:

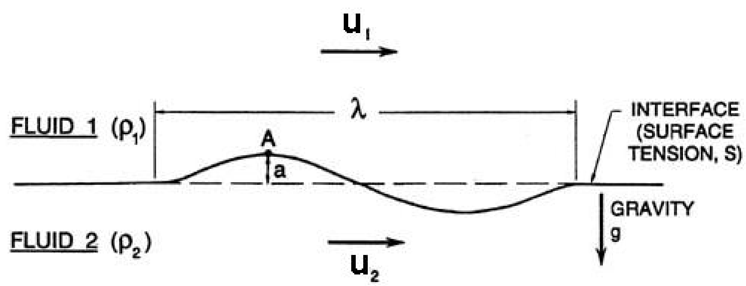

Figure 1: Sketch showing the notation for Kelvin-Helmholtz instability.

- a buoyancy force due to gravity and proportional to the difference in the densities of the two fluids. In a horizontal flow in which the upper fluid is lighter than the lower fluid this force is stabilizing. When the reverse is true the buoyancy force is destabilizing and this causes Rayleigh-Taylor instabilities. When the streams are vertical as in vertical annular flow the role played by the buoyancy force is less clear.
- a surface tension force that is always stabilizing.
- a Bernoulli effect that implies a change in the pressure acting on the interface caused by a change in velocity resulting from the displacement, a, of that surface. For example, if the upward displacement of the point A in figure 2 were to cause an increase in the local velocity of fluid 1 and a decrease in the local velocity of fluid 2, this would imply an induced pressure difference at the point A that would increase the amplitude of the distortion, a.

The interplay between these forces is most readily illustrated by a simple example. Neglecting viscous effects, one can readily construct the planar, incompressible potential flow solution for two semi-infinite horizontal streams separated by a plane horizontal interface (as in figure 1) on which small amplitude waves have formed. Then it is readily shown (Lamb 1879, Yih 1965) that Kelvin-Helmholtz instability will occur when

$$\frac{g\Delta\rho}{\kappa} + \mathcal{S}\kappa - \frac{\rho_1\rho_2(\Delta u)^2}{\rho_1+\rho_2} < 0 \tag{1}$$

where $\mathcal{S}$ is the surface tension of the interface. The contributions from the three previously mentioned forces are self-evident. Note that the surface tension effect is stabilizing since that term is always positive, the buoyancy effect may be stabilizing or destabilizing depending on the sign of $\Delta\rho$ and the Bernoulli effect is always destabilizing. Clearly, one subset of this class of Kelvin-Helmholtz instabilities are the Rayleigh-Taylor instabilities that occur in the absence of flow ($\Delta u = 0$) when $\Delta\rho$ is negative. In that static case, the above relation shows that the interface is unstable to all wave numbers less than the critical value, $\kappa = \kappa_c$, where

$$\kappa_c = \left(\frac{g(-\Delta\rho)}{\mathcal{S}}\right)^{\frac{1}{2}} \tag{2}$$

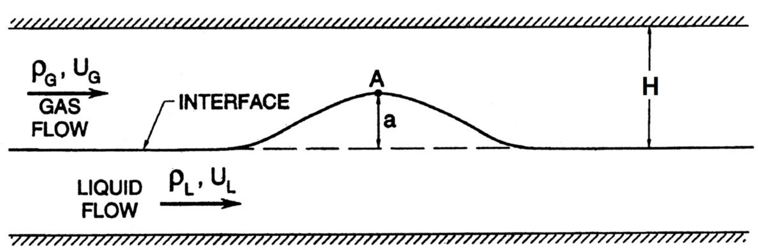

Figure 2: Sketch showing the notation for stratified flow instability.

The Bernoulli effect is frequently the primary cause of instability in a separated flow and can lead to transition to a plug or slug flow regime. As a first example of the instability induced by the Bernoulli effect, consider the stability of the horizontal stratified flow depicted in figure 2 where the destabilizing Bernoulli effect is primarily opposed by a stabilizing buoyancy force. An approximate instability condition is readily derived by observing that the formation of a wave (such as that depicted in figure 2) will lead to a reduced pressure, p_A, in the gas in the orifice formed by that wave. The reduction below the mean gas pressure, $\bar{p}_G$, will be given by Bernoulli's equation as

$$p_A - \bar{p}_G = -\rho_G u_G^2 a/H \tag{3}$$

provided $a \ll H$. The restraining pressure is given by the buoyancy effect of the elevated interface, namely $(\rho_L - \rho_G)ga$. It follows that the flow will become unstable when

$$u_G^2 > gH\Delta\rho/\rho_G \tag{4}$$

In this case the liquid velocity has been neglected since it is normally small compared with the gas velocity. Consequently, the instability criterion provides an upper limit on the gas velocity that is, in effect, the velocity difference. Taitel and Dukler (1976) compared this prediction for the boundary of the stratified flow regime in a horizontal pipe with the experimental observations of Mandhane *et al.* (1974) and found substantial agreement.

As a second example consider vertical annular flow that becomes unstable when the Bernoulli force overcomes the stabilizing surface tension force. From equation 1, this implies that disturbances with wavelengths greater than a critical value, λ_c, will be unstable and that

$$\lambda_c = 2\pi\mathcal{S}(\rho_1 + \rho_2)\big/\rho_1\rho_2(\Delta u)^2 \tag{5}$$

For a liquid stream and a gas stream (as is normally the case in annular flow) and with $\rho_L \ll \rho_G$ this becomes

$$\lambda_c = 2\pi\mathcal{S}\big/\rho_G(\Delta u)^2 \tag{6}$$

Now consider the application of this criterion to a well-developed annular flow at high gas volume fraction in which $\Delta u \approx j_G$. Then for a water/air mixture equation 6 predicts critical wavelengths of 0.4 cm and 40 cm for $j_G = 10\ m/s$ and $j_G = 1\ m/s$ respectively. In other words, at low values of j_G only larger wavelengths are unstable and this seems to be in accord with the break-up of the flow into large slugs. On the other hand at higher j_G flow rates, even quite small wavelengths are unstable and the liquid gets torn apart into the small droplets carried in the core gas flow.