

## 6.4.2 Homogeneous Vaporization

Homogeneous vaporization is identified as vaporization in which the principal source of the latent heat supply to the interface is the liquid itself rather than some nearby heat source. A possible model could be a simplified version of the equations governing the dynamics of a simple spherical bubble of radius,  $R(t)$ . The reader who seeks greater detail is referred to the presentation in Brennen (1995) that includes many of the lesser features omitted here. Lord Rayleigh (1917) first derived the equation governing the radius,  $R(t)$ , of a spherical vapor/gas bubble in a liquid of density,  $\rho_L$ , when the pressure inside the bubble is  $p_B(t)$ , the pressure far away in the liquid is  $p_\infty(t)$ , and the surface tension is  $\mathcal{S}$ , namely:

$$\frac{p_B(t) - p_\infty(t)}{\rho_L} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{2\mathcal{S}}{\rho_L R} \quad (1)$$

The pressure in the bubble,  $p_B(t)$ , may be comprised of a component due to any non-condensable gas present as well as the vapor pressure of the surrounding liquid *at the prevailing temperature in the bubble*,  $T_B(t)$ . The first component, that due to any non-condensable gas, is important but will not be central to the current presentation. On the other hand the vapor pressure, and, in particular, the prevailing temperature in the bubble play a key role in the phenomena that are manifest.

In any liquid volume that is mostly at a temperature close to its triple point, the vapor density is so small that only a very small mass of liquid on the surface of the bubble needs to evaporate in order to supply the increase in bubble volume associated with the bubble growth. Moreover, that small mass of liquid means that only a small supply of heat to the interface is needed to effect the evaporation. And, in turn, that small heat flux only creates a small thermal boundary layer on the bubble surface so that the temperature in the bubble,  $T_B(t)$ , is only very slightly depressed below the prevailing temperature in the bulk of the liquid,  $T_\infty$ .

The converse of this is a liquid that is mostly at a higher temperature, so that the density of the vapor is such that a significant mass of liquid must be vaporized at the bubble surface in order to provide the volume needed for the bubble growth. This implies a substantial heat flux to the interface in order to provide the latent heat for that evaporation; and that heat flux, in turn, usually causes a significant reduction in the temperature of the bubble contents,  $T_B(t)$  (see below for an exception to this consequence). It follows that the vapor pressure in the bubble decreases so that the pressure difference driving the bubble growth, namely  $p_B(t) - p_\infty$ , decreases and, therefore, according to equation 1, the rate of bubble growth decreases. This effect of the liquid temperature in depressing the rate of bubble growth is called the *thermal effect on bubble growth* and it can cause quite a dramatic difference in the resulting bubble dynamics. Perhaps this is most dramatically recognized in the bubble growth in water at normal temperatures. Bubble growth at room temperatures (that are close to the triple point of water) are most frequently observed as

cavitation (see Brennen 1995), a phenomenon in which the growth (and the subsequent collapse) of bubbles is extremely explosive and violent. On the other hand, bubble growth in a pot of boiling water on the stove at  $100^\circ\text{C}$  is substantially inhibited by thermal effects and is therefore much less explosive, much less violent.

These effects can be quantified using the following analyses. First, in the case of no thermal effect, the temperature of the bubble contents will be close to the liquid temperature and therefore the bubble pressure will be roughly constant (neglecting the effect of any non-condensable gas). Then, if the pressures are assumed constant and the surface tension is neglected, equation 1 can be integrated to yield

$$\frac{dR}{dt} = \left[ \frac{2(p_B - p_\infty)}{3\rho_L} \right]^{\frac{1}{2}} \quad \text{and} \quad R = \left[ \frac{2(p_B - p_\infty)}{3\rho_L} \right]^{\frac{1}{2}} t \quad (2)$$

where the integration constant is absorbed into the origin of  $t$ . This result implies explosive bubble growth, with a volume increasing like  $t^3$ ; it is the kind of bubble growth characteristic of cavitation (Brennen 1995).

For contrast, consider the thermally-inhibited growth characteristic of boiling in which the growth is controlled by the rate at which heat can diffuse through an interfacial thermal boundary layer to provide the latent heat of vaporization. The rate of volume growth of the bubble,  $4\pi R^2 dR/dt$ , requires a mass rate of evaporation equal to  $4\pi R^2 (dR/dt)/\rho_V$  where  $\rho_V$  is the vapor density in the bubble. To evaporate this mass requires a rate of heat supply to the interface equal to

$$4\pi R^2 (dR/dt)/(\mathcal{L}\rho_V) \quad (3)$$

where  $\mathcal{L}$  is the latent heat of evaporation. This heat must diffuse through the thermal boundary layer that builds up in the liquid on the bubble surface and causes the bubble temperature,  $T_B$ , to fall below the liquid temperature outside of the boundary layer,  $T_\infty$ . It is this temperature difference,  $(T_\infty - T_B)$ , that drives heat to the bubble surface at a rate given approximately by

$$4\pi R^2 k_L (T_\infty - T_B)/\delta \quad (4)$$

where  $k_L$  is the thermal conductivity of the liquid and  $\delta$  is the thickness of the thermal boundary layer. For growth that begins at time  $t = 0$  this thickness will be given approximately by

$$\delta \approx (\alpha_L t)^{\frac{1}{2}} \quad (5)$$

where  $\alpha_L$  is the thermal diffusivity of the liquid and the thinning of the boundary layer as the bubble grows has been neglected. Furthermore, since  $T_B$  is the temperature of the interface it should be roughly equal to the vapor temperature at the bubble pressure,  $p_B$ , and using the Clausius-Clapeyron relation (Brennen 2005):

$$T_\infty - T_B = \frac{(p_B - p_V)T_B}{\rho_V \mathcal{L}} \quad (6)$$

Equating the expressions 3 and 4 and using the expressions 5 and 6 the following bubble growth rate is obtained:

$$\frac{dR}{dt} = \frac{k_L T_B (p_B - p_V)}{(\alpha t)^{\frac{1}{2}}} \quad \text{and so} \quad R \propto t^{\frac{1}{2}} \quad (7)$$

This rate of growth is much slower than given by the expression 2 and is characteristic of boiling in water at normal pressures.

To summarize, the above analyses (given in much more detail in Brennen 1995) lead, naturally, to two technologically important multiphase phenomena, namely cavitation and boiling. The essential difference is that bubble growth (and collapse) in boiling is inhibited by limitations on the heat transfer at the interface whereas bubble growth (and collapse) in cavitation is not limited by heat transfer but only by the inertia of the surrounding liquid. Cavitation is therefore an explosive (and implosive) process that is far more violent and damaging than the corresponding bubble dynamics of boiling.